Solutions

5.1: Graphing Linear Inequalities

The goal of the next two sections is to begin solving linear programming problems with two unknowns. We use inequalitiees to describe the constraints in a problem, such as limitations on resources.

Inequalities and Manipulations.

(a) $a \leq b$ ($a \geq b$) means that a is less (greater) than or equal to b.

(b) $a < b$ ($a > b$) means that a is strictly less (greater) than b.

(c) The same quantity can be added to or subtracted from both sides of an inequality, just as in the equality case.

(d) The same *positive* quantity can be multiplied or divided from both sides of an inequality, just as in the equality case.

(e) The same *negative* quantity can be multiplied or divided from both sides of an inequality, *but the inequality must be reversed.*

(f) If $a \leq b$, a is less than or equal to b, then $b \geq a$, b is greater than or equal to a.

Example 1.

(a) $0 \leq 1$, $-2 \leq -2$, $99 \geq 3$, $e \leq \pi$, etc.

(b) $0 < 1$, $99 > 3$, and $e < \pi$, but -2 is not strictly less than -2.

(c) If $x \leq y$ then $x - 4 \leq y - 4$.

(d) If $x \leq y$ then $3x \leq 3y$.

(e) If $x \leq y$ then $-3x \geq -3y$.

(f) If $3x + 6 \leq 5y$ then $5y \geq 3x + 6$.

Solving Linear Inequalities. A linear inequality is simply an linear equation where the equals sign (=) is replaced by an inequality sign ($\leq$,$\geq$, $<$, or $>$.) Some examples in one, two, three, and four variables are

$$ax \leq b(\text{or} ax \geq b)$$
$$ax + by \leq c(\text{or} ax + by \geq c)$$
$$ax + by + cz \leq d(\text{or} ax + by + cz \geq d)$$
$$ax + by + cz + dw \leq e(\text{or} ax + by + cz + dw \geq e),$$

where a, b, c, d, and e are real constants.

Example 2. Solve the following inequalities in one variable.

(a) $2x + 8 \geq 89$, $2x \geq 81$, $x \geq \frac{81}{2} = 40.5$

(b) $8 - 4x \leq 40$, $-4x \leq 32$, $x \geq 8$

(c) $2x + 8 \leq 2 - 4x$

$6x \leq -6$

$x \leq -1$

Example 3. Use the provided graph paper to sketch the region represented by the following linear inequalities in one or two variables. Put one inequality on each graph.

See next page for graphs

(a) $3x - 2y \leq 6 \Rightarrow -2y \leq 6 - 3x \Rightarrow y \geq \frac{3x-6}{2}$

(b) $6x \leq 12 + 4y \Rightarrow 6x - 12 \leq 4y \Rightarrow y \geq \frac{6x-12}{4} = \frac{3x-6}{2}$

(c) $x \leq -1$

(d) $y \geq 0$

(e) $x \geq 3y \Rightarrow y \leq \frac{x}{3}$

Example 4. Use the provided graph paper to sketch the region represented by the following systems of linear inequality. Put one system on each graph and clearly indicate the intersection points.

(a) $2x - 5y \leq 10$, $x + 2y \leq 8 \Rightarrow y \geq \frac{2x-10}{5}$, $y \leq \frac{8-x}{2}$

(b) $3x - 2y \leq 6$, $x + y \geq 6$, $y \leq 4 \Rightarrow y \geq \frac{3x-6}{2}$, $y \geq 6-x$, $y \leq 4$

(c) $2x + 4y \geq 12$, $x \leq 5$, $y \leq 3$, $x \geq 0$, $y \geq 0 \Rightarrow y \geq \frac{12-2x}{4}$, $x \leq 5$, $y \leq 3$, $x \geq 0$, $y \geq 0$

Example 5. Socaccio Pistachio Inc. makes two types of pistachio nut: Dazzling Red and Organic. Pistachio nuts require food color and salt, and the following table shows the amount of food color and salt required for a 1-kilogram batch of pistachios, as well as the total amount of these ingredients available each day.

	Dazzling Red	Organic	Total Available
Food Color (g)	2	1	20
Salt (g)	10	20	220

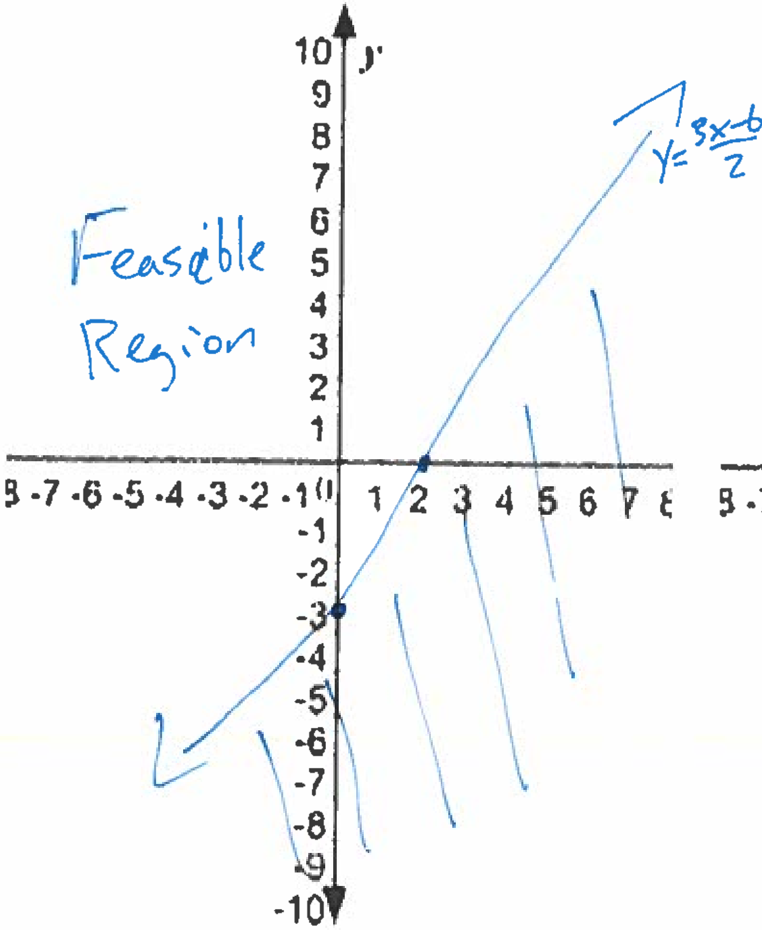
Use a graph to show the possible numbers of batches of each type of pistachio Socaccio can produce a day. This region (the solution set of a system of inequalities) is called the feasible region.

$2x + y \leq 20 \Rightarrow y \leq 20 - 2x$

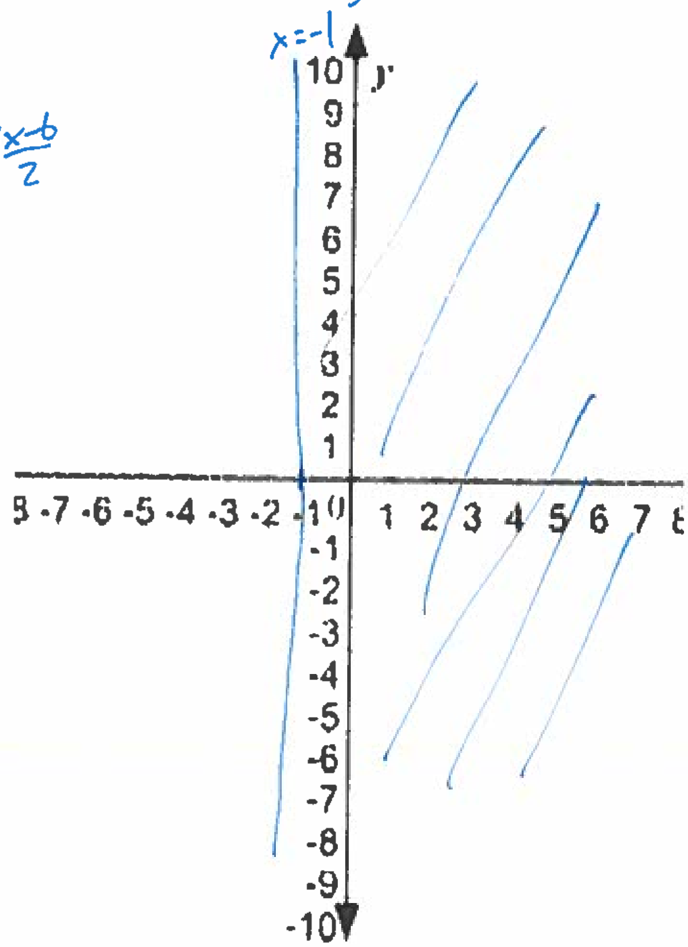
$10x + 20y \leq 220 \Rightarrow y \leq \frac{22-x}{2}$

Implicit Equations: $x \geq 0$, $y \geq 0$

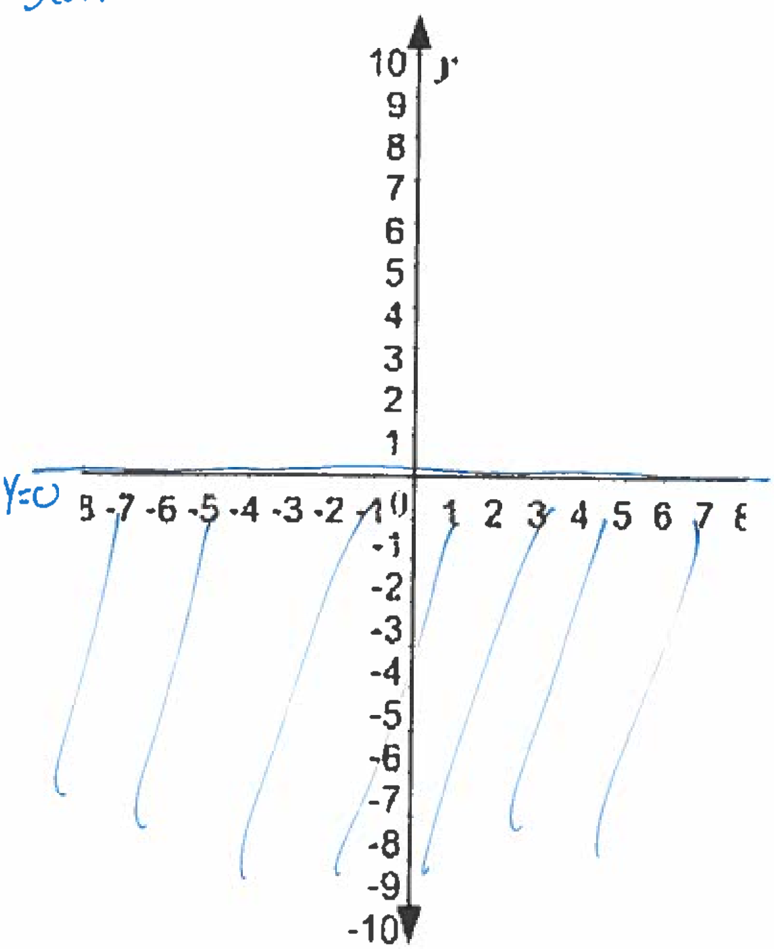
3(a) and 3(b)



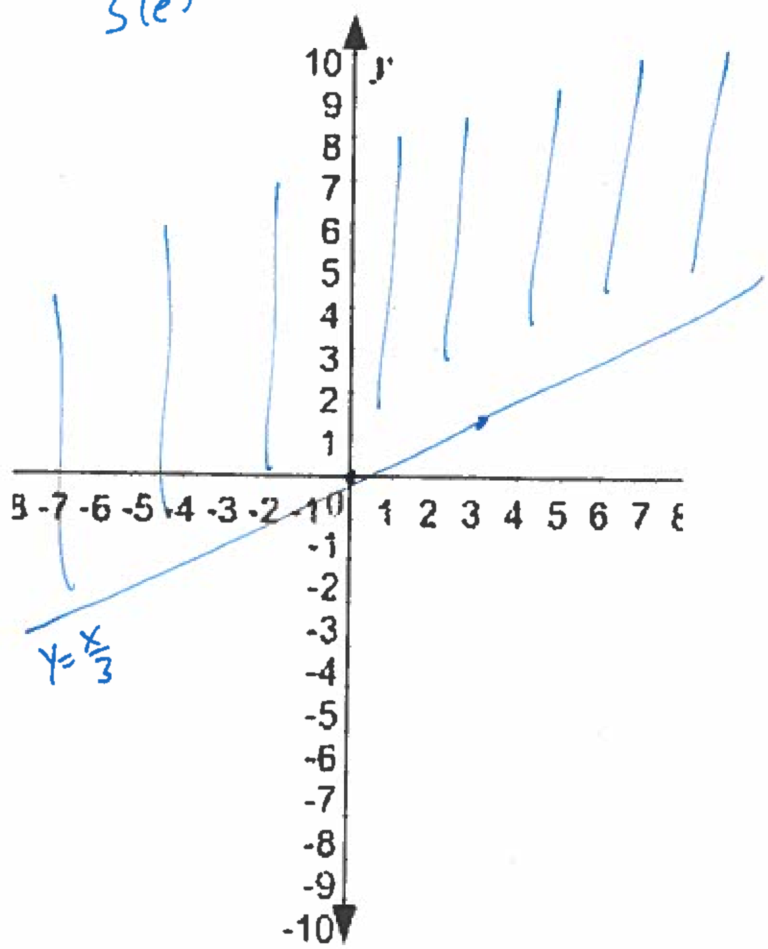
3(c)



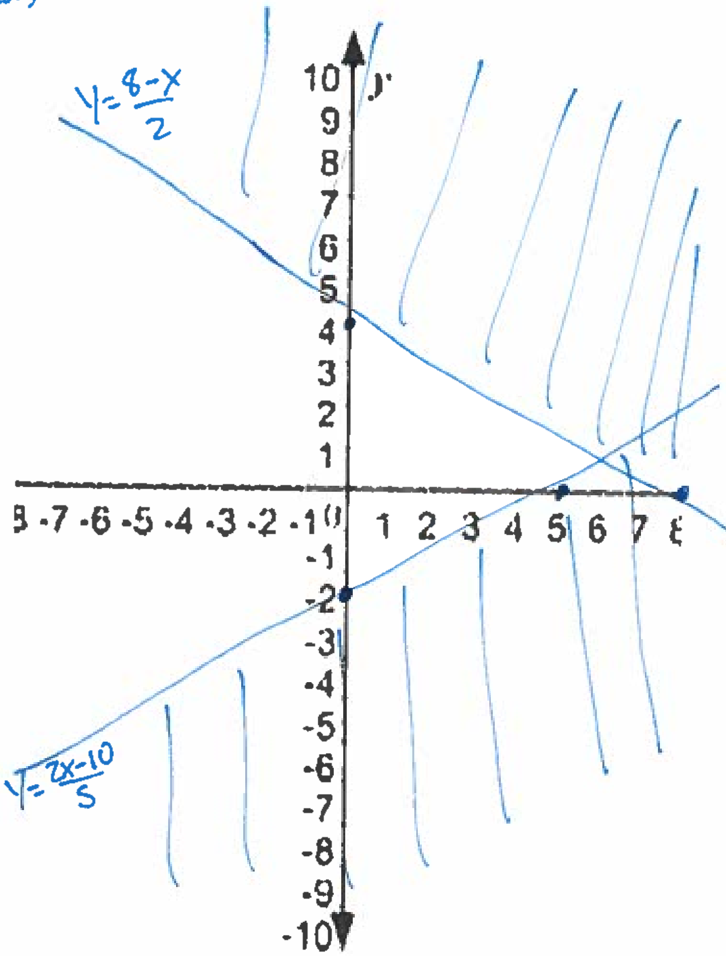
3(d)



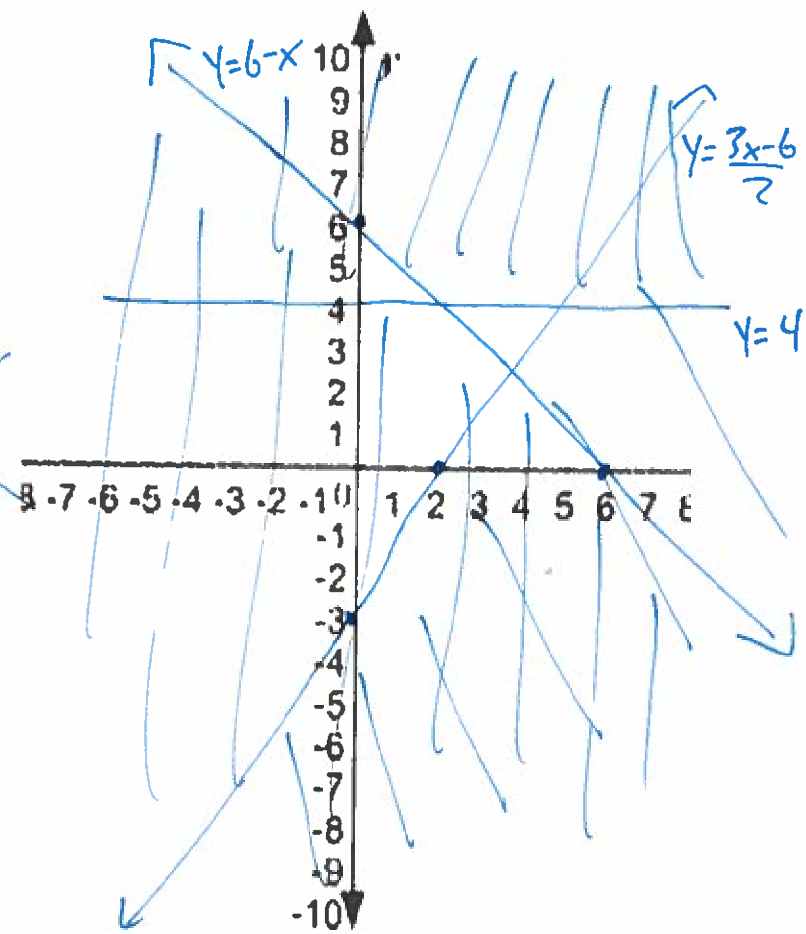
3(e)



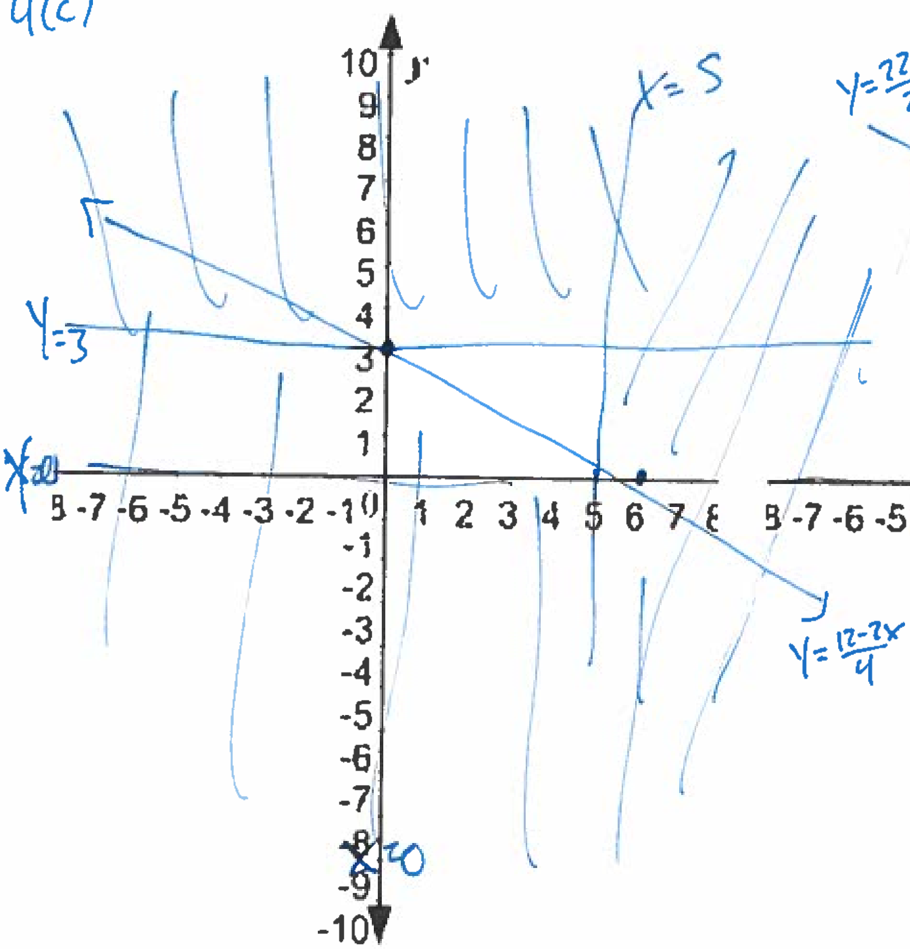
4(a)

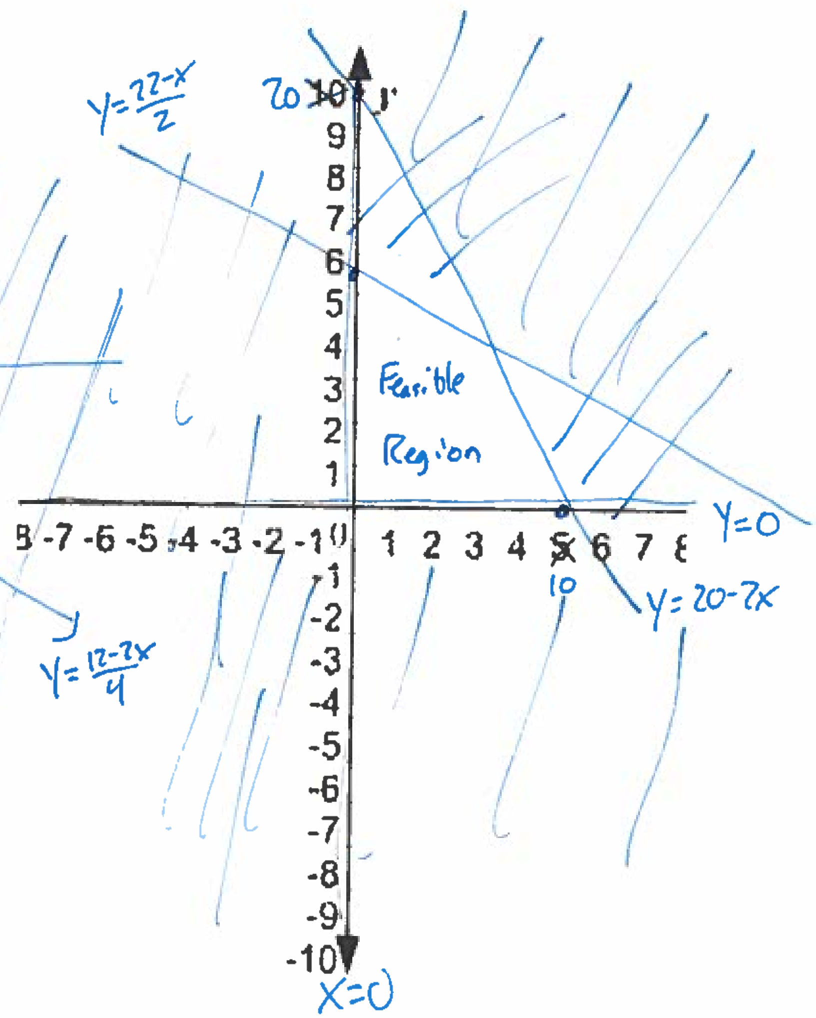

y = (8-x)/2
y = (2x-10)/5

4(b)


y=6-x
y = (3x-6)/2
y= 4

4(c)


y=3
x= 5
y = (12-2x)/4
x=0
x=0


y = (22-x)/2
20
Feasible Region
y=0
y= 20-2x
10
x=0